Math 99 – Factoring Review

Common factors

This is normally the simplest situation in which we factor terms, to factor theses expressions you look for the common factors that belong to all the terms, you then "pull out" these common factors and put the unused parts in the parentheses.

Example 1: Factor $4t^3 - 6t$

Solution: The two terms of this expression are $4t^3$ and $-6t$
the common factors are 2 and t so the common factor is 2t.

Thus $4t^3 - 6t = \underline{2t(2t^2 - 3)}$

Example 2: Factor the expression $3x^3y^2 + 9x^2y^3$

Solution: The 2 terms of this expression are $3x^3y^2$ and $9x^2y^3$
The common factors are 3, x^2 and y^2 so combined the common factor is $3x^2y^2$.

So $3x^3y^2 + 9x^2y^3 = \underline{3x^2y^2(x + 3y)}$

Example 3: Factor the expression $3ab^2 + 6a^2b^2 - 12a^3b^3$

Solution: The 3 terms of this expression are $3ab^2$, $6a^2b^2$ and $-12a^3b^3$
The common factors are 3, a and b^2 so combined the common factor is $3ab^2$.

So $3ab^2 + 6a^2b^2 - 12a^3b^3 = \underline{3ab^2(1 + 2a - 4ab)}$

Example 4: Factor the expression $4x^2y - 8xy + 10x$

Solution: The 3 terms of this expression are $4x^2y$, $-8xy$ and $10x$
The common factors are 2 , and x so combined the common factor is 2x.

So $4x^2y - 8xy + 10x = \underline{2x(2xy - 4y + 5)}$

Difference of squares

In this situation the expression will always contain two terms that are subtracted. The two terms will in turn be squares of some other term. When you factor such expressions the answer is always in the form $(a + b)(a - b)$ where a and b are the square roots of the first and second terms.

Example 5: Factor the expression $t^2 - 4$

Solution: The 2 terms of this expression are t^2 and 4, their square roots are t and 2.

So $t^2 - 4 = \underline{(t + 2)(t - 2)}$

Example 6: Factor the expression $25x^2 – 16$

Solution: The 2 terms of this expression are $25x^2$ and 16, their square roots are 5x and 4.

So $25x^2 – 16 = \underline{(5x + 4)(5x – 4)}$

Example 6: Factor the expression $4y^6 – 9a^4$

Solution: The 2 terms of this expression are $4y^6$ and $9a^4$, their square roots are $2y^3$ and $3a^2$.

So $4y^2 – 9a^2 = \underline{(2y^3 + 3a^2)(2y^3 – 3a^2)}$

Simple Trinomials

These expressions contain three terms (hence the name trinomial). The most common type ***are simple trinomials*** – these are expressions where there is only an x^2 term and have the following general form $x^2 + bx + c$. When you factor $x^2 + bx + c$ it will always take the form (x …..)(x ……). The missing terms in the parenthesis are the two numbers which are the factors of the constant term c and who's sum equals the coefficient of x called b.

Example 7: Factor the expression $x^2 + 5x + 6$

Solution : In this situation we are looking for two numbers that multiply together to give 6 and when you add them you get 5.

So $x^2 + 5x + 6 = \underline{(x + 2)(x + 3)}$

Example 8: Factor the expression $x^2 – 6x + 8$

Solution: In this situation we are looking for two numbers that multiply together to give 8 and when you add them you get – 6.

So $x^2 – 6x + 8 = \underline{(x – 2)(x – 4)}$

Example 9: Factor the expression $x^2 + 2x – 15$

Solution : In this situation we are looking for two numbers that multiply together to give – 15 and when you add them you get 2.

So $x^2 + 2x – 15 = \underline{(x + 5)(x – 3)}$

Example 10: Factor the expression $y^2 - y - 12$

Solution: In this situation we are looking for two numbers that multiply together to give – 12 and when you add them you get – 1.

So $y^2 - y - 12 \quad = \quad \underline{(y + 3)(y - 4)}$

Complex Trinomials

A complex trinomial is one in which typically there is not single x^2 term but some multiple instead and it has the general form $ax^2 + bx + c$. This handout describes the guess-and-check method. When you factor a complex trinomial of the form $ax^2 + bx + c$ there are 3 steps.

1. Get the possible x term parts of the factor.
2. Get the possible constant term parts of the factor.
3. Determine which combination of the above, if any, works.

Example 11: Factor the expression $2x^2 + 5x - 3$

Solution: Step 1 **Possible x-terms**. There are a set of rules you can follow that will help you to factorise a complex trinomial expression the first is look at the number of x^2 the expression contains , this will tell you the possible number of x's that go into the factor , so for example if the expression has $2x^2$ can only be formed by 2x times x. So the solution must take the form on the other hand if it were $4x^2$ it could be one of two possible situations either **(**4x .....)(x ......) or (2x .....)(2x ......). In this example we have $2x^2 + 5x - 3$ and we get.

$$2x^2 + 5x - 3 \quad = \quad (2x \,.....)(x \,......)$$

Step 2 **Possible constant terms.** The second stage in the process is to look at the constant term, in this case its value is – 3 and determine all of the possible factor pairs. So in this example we can use (1 and – 3) and (– 3 and 1) and (– 1 and 3) and (3 and – 1)

Step 3. **Which one works?** The 4 combinations are shown below. We FOIL out each one until we get the one that works. Notice that in the factors of – 3 the order is important.

$(2x + 1)(x - 3)$	this gives $2x^2 - 5x - 3$	
$(2x - 1)(x + 3)$	this gives $2x^2 + 5x - 3$	This is the correct one.
$(2x + 3)(x - 1)$	this gives $2x^2 + x - 3$	
$(2x - 3)(x + 1)$	this gives $2x^2 - x - 3$	

Only one combination actually works and this is our solution.

So $2x^2 + 5x - 3 = \underline{(2x - 1)(x + 3)}$

Example 12: Factor the expression $5y^2 - 12y + 4$

Solution: Step 1 **Possible x-terms**. There are a set of rules you can follow that will help you to factorise a complex trinomial expression the first is look at the number of y^2 the expression contains , this will tell you the possible number of y's that go into the factor , so for example since $5y^2$ can only be formed by 5y times y. So the solution must take the form

$$5y^2 - 12y + 4 \quad = \quad (5y \,.....)(y \,......)$$

Step 2 **Possible constant terms.** The second stage in the process is to look at the constant term, in this case its value is + 4, so in this example we can use (1 and 4) and (4 and 1) and (2 and 2) and (– 1 and – 4) and (– 4 and – 1) and (– 2 and –2).

Step 3. **Which one works?** The 6 combinations are shown below. We FOIL out each one until we get the one that works.

$(5y + 1)(y + 4)$	this gives $5y^2 + 21y + 4$	
$(5y + 4)(y + 1)$	this gives $5y^2 + 9y + 4$	
$(5y + 2)(y + 2)$	this gives $5y^2 + 12y + 4$	
$(5y - 1)(y - 4)$	this gives $5y^2 - 21y + 4$	
$(5y - 4)(y - 1)$	this gives $5y^2 - 9y + 4$	
$\underline{(5y - 2)(y - 2)}$	this gives $5y^2 - 12y + 4$	This is the correct one.

Notice that the first three choices were bound to give positive values for the y-term and so we could have saved ourselves some time by not bothering to check them as they clearly cannot get us the – 12y term that we need.

Combined factoring.

In this situation we combine two or more of the methods explained earlier.

Example13: Factor the expression $5x^2 - 180$

Solution: First we take a common factor of 5 from the two terms then we factor the difference of Two squares that remains.

So	$5x^2 - 180$	=	$5(x^2 - 36)$	Take out a common factor of 5
	$5(x^2 - 36)$	=	$5(x + 6)(x - 6)$	factorise the difference of squares
	$5x^2 - 180$	=	$\underline{5(x + 6)(x - 6)}$	

Example 14: Factor the expression $2ax^2 + 12ax + 18a$

Solution: First we take a common factor of 2a from the 3 terms then we factor what remains.

So	$2ax^2 + 12ax + 18a$	=	$2a(x^2 + 6x + 9)$	Take out a C.F. of 2a
		=	$2a(x + 3)(x + 3)$	Factorise the quadratic
		=	$2a(x + 3)^2$	
		=	$\underline{2a(x + 3)^2}$	

Sum and Differences of Cubes

You may, on occasion, encounter a sum or difference of two cubes. By multiplying the factors, we may check that

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

and

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Using these formulas, we may factor sums and differences of cubes.

Example 15: Factor the expression $t^3 - 8$

Solution: This is a difference of cubes and the 2 terms of this expression are t^3 and 8. Their cube roots are t and 2.

So $t^3 - 8 = \underline{(t - 2)(t^2 + 2t + 4)}$

Example 16: Factor the expression $y^3 + 27$

Solution: This is a sum of cubes and the 2 terms of this expression are y^3 and 27. Their cube roots are y and 3.

So $y^2 + 27 = \underline{(y + 3)(y^2 - 3y + 9)}$